3.4 Right Triangle Trigonometry

A solution to the equation $\sin\alpha = \frac{1}{2}$ is an angle whose sine is $\frac{1}{2}$. Because $\sin 30° = \frac{1}{2}$ and $\sin 150° = \frac{1}{2}$, α could be 30° or 150°. Since any angle with the same terminal side as 30° or 150° is also a solution, there are infinitely many solutions. Since right triangles have only acute angles, we are only interested in the acute solutions in this section.

Examples:
Find the angle α that satisfies each equation where $0° \le \alpha \le 90°$.

a. $\sin\alpha = \sqrt{3}/2$ b. $\cos\alpha = 1$ c. $\tan\alpha = 1$

Inverse Sine, Cosine, and Tangent Functions
To calculate the size of angles with a given sine, cosine, or tangent, we use the inverse trigonometric functions $\sin^{-1} x$, $\cos^{-1} x$, and $\tan^{-1} x$, also known as arcsine (arcsin), arccosine (arccos), and arctangent (arctan).

★ **The -1 in $\sin^{-1} x$ does not indicate a reciprocal. $\sin^{-1} x \ne \dfrac{1}{\sin x}$. The -1 indicates an inverse function. $\sin^{-1} x$, $\cos^{-1} x$, and $\tan^{-1} x$ are angles!**

Because there are infinitely many angles that have a given sine, cosine, or tangent, we define the inverse functions precisely by restricting their domains:

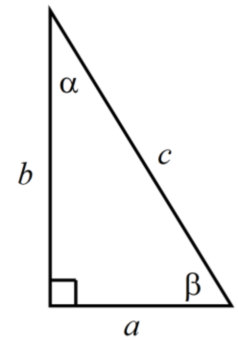
- The inverse sine of x ($\sin^{-1} x$ or $\arcsin x$) is the angle between $-90°$ and $90°$ whose sine is x.
  - If $\sin\alpha = x$, and $-90° \le \alpha \le 90°$, then $\alpha = \sin^{-1} x$.
- The inverse cosine of x ($\cos^{-1} x$ or $\arccos x$) is the angle between $0°$ and $180°$ whose cosine is x.
  - If $\cos\alpha = x$, and $0° \le \alpha \le 180°$, then $\alpha = \cos^{-1} x$.
- The inverse tangent of x ($\tan^{-1} x$ or $\arctan x$) is the angle between $-90°$ and $90°$ whose tangent is x.
  - If $\tan\alpha = x$, and $-90° < \alpha < 90°$, then $\alpha = \tan^{-1} x$.

Examples:
Evaluate each expression. Give the result in degrees. Where necessary, round to the nearest tenth.

a. $\cos^{-1}\left(\sqrt{2}/2\right)$ b. $\arcsin\left(\sqrt{3}/2\right)$ c. $\tan^{-1}(6.1)$

Solving Right Triangles

Finding all the missing angle measures and side lengths of a triangle is called "solving a triangle". In a right triangle, we usually name the acute angles α and β (beta) and the lengths of the sides opposite those angles a and b, respectively. The 90° angle is γ (gamma) and the length of the side opposite the right angle (the hypotenuse) is c.



- If you know the lengths of two of the sides, use the Pythagorean Theorem to find the length of the third side.
- If you know the measure of one of the acute angles, use the fact that the angles in a triangle add to 180° to find the measure of the other angle.
- If you know the measure of one angle and the length of one side, use sin, cos, or tan to figure out the lengths of the other sides.
- If you know the lengths of the sides and need to figure out the angle measures, use inverse functions ($\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$).

Examples:

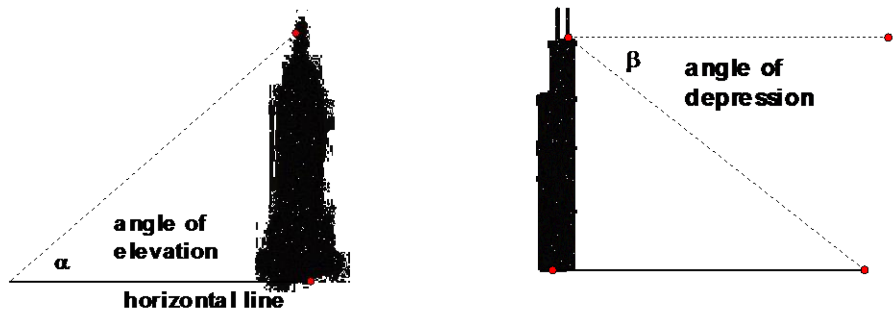
Solve the right triangle in which $\alpha = 60°$ and $c = 2$.

Solve the right triangle in which $a = 2$ and $b = 5$.

Solve the right triangle in which $\beta = 20°$ and $b = 15$.

Solve the right triangle in which $a = 5$ and $c = 13$.

Using trigonometry, we can find the size of an object without actually measuring the object. Two common terms used in this regard are **angle of elevation** and **angle of depression**.



Examples:
The angle of elevation of the top of a cell phone tower is 38.2° at a distance of 344 feet from the tower. What is the height of the tower?

At one location, the angle of elevation of the top of an antenna is 44.2°. At a point that is 100 feet closer to the antenna, the angle of elevation is 63.1°. What is the height of the antenna?